Environmental Catastrophes and Non-Expected Utility Maximization: An Experimental Evaluation

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ABSTRACT

Traditional benefit/cost analyses of environmental hazards typically compare the expected costs and benefits of various policies, an approach that implicitly assumes agents maximize expected utility. While the expected utility (EU) framework is analytically expedient, environmental catastrophes imply losses, which have the potential to be quite dramatic. An abundance of experimental evidence has called the empirical validity of EU into question for non-market choices, especially when people confront losses from the status quo. This paper produces evidence on preferences regarding uncertain outcomes that include the potential, with small probability, for a large loss to occur. We conduct a battery of experiments in which people confront a set of risky choices, or "lotteries." Because environmental catastrophes imply potentially large losses, the lotteries involve amounts of money that people are likely to regard as large. We use their choices to estimate a representation of preferences, which can be thought of as a utility function over lotteries. Under the EU approach, this representation would be linear in the probabilities; the representation is non-linear if the subject does not maximize expected utility. We find that non-linearities are statistically and economically important for a substantial percentage of our subjects. Indifference curves over lotteries are often concave.

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1. Introduction

Many important environmental issues involve the risk of catastrophe—low probability/high severity events like climate change, oil spills, nuclear accidents, and species extinctions (e.g., Nordhaus, 1993; Rosenzweig and Parry, 1994; Brown and Shogren, 1998; Weitzman, 1998). The challenge for policymakers is to frame the debate thoughtfully: how should society manage environmental risks when an enormously costly outcome may occur with small probability? The current approach for dealing with such environmental hazards typically compares the expected costs and benefits of various policies, an approach which implicitly assumes that people maximize expected utility (e.g., Chichilnisky and Heal, 1993).

While any movement toward balancing costs and benefits in policy decisions is worthwhile, the analytically convenient expected utility (EU) model has come under increasing suspicion for many non-market choices, both on a theoretical level and an empirical level, especially for low probability/high severity events. An abundance of experimental evidence calls into question the empirical validity of the expected utility maximization paradigm (see Machina, 1987; Viscusi, 1992; Thaler, 1992; Camerer, 1995). Much of this evidence points to a particularly strong tendency for the expected utility model to fail when the uncertain events include an outcome that is relatively unlikely to occur, but has large payoff implications (see for example Lichtenstein et al., 1978; Baron, 1992). People often use a heuristic when dealing with choices under uncertainty—they focus on probability and consequences separately rather than in combination as predicted by expected utility theory, or most generalized utility theories (e.g., Slovic and Lichtenstein 1968; Schoemaker, 1989). Machina (1982) argues that the analytical convenience of EU is not a compelling reason to maintain the traditional analytic view, since much of the analysis of behavior under risk goes through, albeit in slightly more complicated form, when the expected utility maximization hypothesis is dropped.

While these points suggest a need to revise the way society looks at policy towards environmental catastrophes, much of the existing experimental evidence has been based on subject behavior when confronted with uncertainties over potential *good* outcomes. But evidence that people may regard gains and losses differently is fairly compelling for non-market choices (Camerer, 1995; Thaler, 1992). We often observe divergence between willingness to pay and willingness to accept in experiments without endogenous market-clearing price feedback; the former reflecting a valuation of gains, and the latter a valuation of losses (see for instance Kahneman et al. 1990). Prospect theory explicitly points to differences in risk attitudes between gains and losses (Tversky and Kahneman, 1981).

Since environmental catastrophes are associated with losses, we believe an analysis that characterizes peoples' preferences over potentially deleterious risky outcomes would yield useful insight to help frame environmental policy discussions on climate change (see for instance Shogren, 1999). This paper provides evidence from experiments designed to test the empirical accuracy of the expected utility model over losses. In contrast to earlier experimental work, our experiments confront subjects with choices between a pair of risky choices, or "lotteries," over potential losses. Our results allow us to infer subjects' preferences regarding uncertain outcomes that include the potential, with small probability, for a large loss to occur.

We use the data to estimate parameters in a representation of individual preferences. This representation can be thought of as a utility function over lotteries. Under the expected utility approach, the representation is linear in the probabilities; under the non-expected utility approach, the representation is non-linear in the probabilities (Machina, 1982). We find that non-linearities are important, both statistically and economically. Subject behavior indicates the probability of the worst possible event enters into the preference representation in a non-linear fashion—indifference curves over lotteries are often concave.

2. Background

We begin with a brief discussion on the traditional expected utility model and violations of the underlying assumptions. The basic idea of the EU model is that if an agent's preferences satisfy three axioms – ordering, continuity, and independence – then her behavior can be modeled as if she is maximizing expected utility. Ordering implies preferences are complete and transitive. Continuity means that if lottery R is preferred to lottery S, and S is preferred to lottery T, then there are real numbers α and β between zero and one such that $\alpha R + (1-\alpha)T$ is preferred to S, while S is preferred to $\beta R + (1-\beta)T$. Independence means that if two lotteries R and S are equally liked by the agent, the gamble composed of a *p* chance of R or a 1-*p* chance of T must be equally liked as the gamble composed of a *p* chance of S or a 1-*p* chance of T (see Machina, 1982; Starmer, 2000).

Suppose that there are three consequences or events. Let y_1, y_2, and y_3 represent the monetary magnitudes of the events, where $y_1 < y_2 < y_3$. That is, the first possible outcome is the worst event, while the third outcome is the best event. Let p_i reflect the probability that outcome y_i will be realized, for i = 1, 2, or 3. Then the lottery **p** is the vector of probabilities (p_1,p_2,p_3). The expected utility hypothesis says that there is an increasing function $u(\bullet)$ over wealth, typically called the von Neumann – Morgenstern utility function, such that the an agent prefers lottery **p** to lottery **q** if and only if $V(\mathbf{p}) > V(\mathbf{q})$, where

$$V(\mathbf{p}) = \sum_{i=1}^{3} u(y_i)p_i \ . \qquad (1)$$

The function $V(\bullet)$ is called the expected utility representation. Since the three probabilities must sum to one, eq. (1) can be simplified to

$$V(\mathbf{p}) = [u(y_1) - u(y_2)]p_1 + [u(y_3) - u(y_2)]p_3. \quad (2)$$

Note the values $u(y_i)$ are constants, once the magnitudes of the outcomes are specified. Correspondingly, the representation $V(\bullet)$ is linear in the probabilities. Since $y_1 < y_2 < y_3$ and $u(\bullet)$ is increasing in y, the coefficient on p_1 is negative, while the coefficient on p_3 is positive.

One can plot level curves for the representation in eq. (2) using a two-dimensional diagram. An example of this plot, which is known as the Marschak-Machina triangle (Marschak, 1950), is provided in Figure 1. The slopes of indifference curves within this diagram can be found by implicitly differentiating (2) to get

$$0 = dV = [u(y_1) - u(y_2)]dp_1 + [u(y_3) - u(y_2)]dp_3$$

$$\leftrightarrow dp_3/dp_1 = -[u(y_1) - u(y_2)]/ [u(y_3) - u(y_2)]. \quad (3)$$

Since the $u(y_i)$ are constants, dp_3/dp_1 is a constant. That is, indifference curves are parallel straight lines. This is a testable hypothesis.

The Allais Paradox is a well-known example of violation of parallel linear indifference curves. As initially presented (Allais, 1953), an individual has to choose between a_1 or a_2 and between a_3 or a_4, where a_1, a_2, a_3, and a_4 are represented as

a_1:	1.00 chance of $1,000,000;	a_2:	.10 chance of $5,000,000,
			.89 chance of $1,000,000,

			.01 chance of \$0;
a_3:	.10 chance of \$5,000,000,	a_4:	.11 chance of \$1,000,000,
	.90 chance of \$0;		.89 chance of \$0.

Notice that the lines connecting a_1 to a_2 and a_3 to a_4 are parallel. If an agent's indifference curves are straight lines, then he would prefer a_1 to a_2 *and* a_3 to a_4, or he would prefer a_2 to a_1 *and* a_4 to a_3. However, laboratory experimentation has shown that subjects usually choose a_1 and a_3, so that indifference curves cannot be parallel straight lines.[1]

While these earlier experimental results are intriguing, virtually all of this evidence is based on lotteries over gains. Thaler (1992) cites an abundance of evidence that suggests individuals regard losses differently from gains. Since most important environmental consequences entail lotteries over losses, it is not transparent that the extant experimental results have any meaningful implications for environmental economics. By contrast, the experiments we design and analyze below are based on lotteries over losses. By analyzing the pattern of subjects' choices, we are able to determine whether their choices are consistent with the expected utility representation.

3. Experimental Design

We present each subject in the experiments with 40 pairs of lotteries, which we called "options." Lotteries are defined as follows. Let x_1, x_2, and x_3 represent the magnitudes of the three consequences or losses, where $x_1 > x_2 > x_3$. The first possible outcome entails the largest loss, while the third outcome entails the smallest loss. With an initial endowment of y_0, these events induce wealth levels $y_i = y_0 - x_i$, I = 1, 2, or 3. In the experiments we report on herein, y_0 = \$100, x_1 = \$80, x_2 = \$30, and x_3 = \$0. Let p_i reflect the probability that outcome x_i will be realized, for i = 1, 2, or 3, and denote the vector of probabilities by $\mathbf{p} = (p_1,p_2,p_3)$. A person's preference ordering over lotteries implies a representation $V(\mathbf{p})$, with this being linear under expected utility.

We build the set of lotteries around three reference lotteries, which we selected to reflect specific environmental risk scenarios. In lottery A, the 'less bad' outcome obtains with a small probability. This describes a situation where both the worst outcome and the less bad outcome are not very likely to occur. In lottery B, the less bad outcome is more likely than the other events, but still is not highly probable. This corresponds to a situation with a substantial chance of medium size losses. In lottery C, losses are quite likely, but they are overwhelmingly more likely to be modest than large. These different scenarios are suggestive of different types of potential catastrophes. For example, while oil spills are not rare, when they occur the damages are usually not enormous (as in lottery B). By contrast, one might argue that while large or enormous damages from global climate change are fairly possible, modest damages are the more likely outcome (as in lottery C).

Figure 1 illustrates our method for selecting lotteries. The three probabilities for lottery A in this example are $p_1 = .05$, $p_2 = .35$ and $p_3 = .6$. The three probabilities for B are $p_1 = .05$, $p_2 = .55$, and $p_3 = .4$. The three probabilities for C are $p_1 = .05$, $p_2 = .75$, and $p_3 = .2$. Notice that in each of these three lotteries, the probability of the worst event (lose $80) is quite small. Each of these reference lotteries was compared to twelve other points; four where p_1 was reduced to .01, four where p_1 was increased to .1, and four where p_1 was increased to .2. The decrease in p_1 from .05 to .01 was combined with a decrease in p_3. Conversely, the increase in p_1 from .05 to either .1 or .2 was combined with an increase in p_3. The decreases (and increases) in p_3 followed a specific path. For example, the four points where p_1 was increased from .05 to .1 are labeled as points B1 (.1,.49,.41), B2 (.1,.45,.45), B3 (.1,.4,.5), and B4 (.1,.3,.6) (note that the figure is not drawn to exact scale).

The actual experimental design follows a five stage procedure:

Stage #1: Starting the Experiment: We recruited subjects from classes at the University of Wyoming, and from the city of Laramie. This allows us to gauge the influence of education level upon observed behavior. They are asked to report to a specified room at a specified time. At that time, the room is closed, and the experiment begins. The subjects take a seat and are given the experimental instructions (see Appendix 1). The monitor reads the instructions, while subjects follow along on their copy. Subjects are told that no communication between subjects will be allowed during the experiment, that anyone who fails to follow the instructions will be asked to leave and forfeit any moneys earned, and that anyone can leave the experiment at any time without prejudice. After the reading of the instructions, any questions are taken. Subjects then fill out a survey that asks their gender, birthdate, highest level of school completed, courses taken in Mathematics, and the subject's personal annual income and his or her families' annual income (see Appendix 2). They are also asked to sign a waiver.

Stage #2: The Option Sheet: After each subject turns in their waiver and the survey, the experiment begins. Each subject starts with a \$100 endowment, and his or her choices and chance affect how much of this money he or she can keep as take-home earnings. Each subject is given an *option sheet* with 40 pairs of options (see Appendix 3). Each option is divided into 3 probabilities:

p_1 is the probability of losing \$80;
p_2 is the probability of losing \$30; and

p_3 is the probability of losing \$0.

For example, if an option has p_1 = 20%, p_2 =50% and p_3 = 30%, this implies a subject has a 20% chance to lose \$80, a 50% chance to lose \$30, and a 30% chance to lose \$0. (These events correspond to taking home \$20, \$70, or \$100.) For each option, the three probabilities always add up to 100% ($p_1 + p_2 + p_3$ = 100%). On the option sheet, each subject must circle his or her preferred option for each of the 40 pairs.

Stage #3: The Tan Pitcher: After filling out the option sheet, all subjects wait until the monitor calls him or her to the front of the room. When called, the subject brings the waiver form, survey, and option sheet. There is be a tan pitcher containing 40 chips on the front table, numbered from 1 to 40. The numbers on the chips correspond to the 40 options on the option sheet. The subject reaches into the tan pitcher without looking at the chips, and picks out a chip. The number on the chip determines which option he or she will play to determine his or her take-home earnings. For example, if he draws chip #23, he plays the option he circled for the pair #23 on his option sheet.

Stage #4: The Blue Pitcher: After the option to be played has been determined, the subject then draws a different chip from a blue pitcher. The blue pitcher has 100 chips, numbered 1 to 100. The number on the chip determines the actual outcome of the option – a loss of either \$80, \$30, or \$0. For example, suppose the option to be played has

$$p_1 = 10\%,\ p_2 = 50\%,\ p_3 = 40\%.$$

If the chip drawn by the subject is numbered between 1 and 10, event 1 obtains, so that the subject loses \$80; if he picks a chip between 11 and 60, he loses \$30; or if he picks a chip between 61 and 100, he loses \$0. If instead, the option to be played has

$$p_1 = 20\%,\ p_2 = 20\%,\ p_3 = 60\%.$$

and the subject draws a chip numbered between 1 and 20, he loses \$80; if he draws a chip between 21 and 40, he loses \$30; if he draws a chip between 41 and 100, he loses \$0.

Stage #5: Ending the experiment: After playing the option, each subject fills out a tax form. After the monitor receives the tax form and the survey form, the subject is paid his or her take-home earnings in cash. The subject then leaves the room.

While subjects necessarily leave the experiment with positive payoffs, endowing them with \$100 at the start makes them see the events as potential losses. Indeed, our observation of subject reaction when the lottery is played indicates that they regard the possibility of losing \$80 as a very bad outcome. All told, 53 subjects participated in our experiments, with the typical subject earning between \$70 and \$75.

4. Econometric Results

To extract information on subjects' preferences over lotteries containing bad outcomes, we applied the following econometric analysis. As noted above, the preference ordering for a certain agent k can be represented by a function $V_k(\mathbf{p})$, where **p** is a probability distribution that places probability p_i on event i = 1, 2 and 3. Agent k prefers lottery **p** to lottery **q** if $V_k(\mathbf{p}) > V_k(\mathbf{q})$. Allowing for decision errors, we regard this choice as probabilistic (Loomes, Moffat, and

Sugden, 1997): agent k chooses lottery **p** over lottery **q** if $V_k(\mathbf{p}) - V_k(\mathbf{q}) + \varepsilon > 0$, where ε reflects decision errors. Accordingly, the probability that agent k chooses lottery **p** over lottery **q** is

$$PR[\varepsilon > V(\mathbf{q}) - V(\mathbf{p})], \tag{4}$$

where 'PR(E)' means "the probability that event E occurs". Once a distribution for ε is specified and a parametric form for V_k is chosen, estimation of the parameters in V_k follows straightforward maximum likelihood techniques (Fomby, Hill, and Johnson, 1988).

Since we are interested in identifying the importance of non-linear effects, a natural approach to take is to specify V_k as a quadratic function. This may be regarded as a second-order Taylor's series approximation to a more general non-linear form. We parameterize the quadratic as:

$$V(\mathbf{p}) = \alpha + \beta_1 p_1 + \beta_2 p_3 + \beta_3 p_1^2 + \beta_4 p_1 p_3 + \beta_5 p_3^2. \tag{5}$$

Let $Y_1 = q_1 - p_1$, $Y_2 = q_3 - p_3$, $Y_3 = q_1^2 - p_1^2$, $Y_4 = q_1 q_3 - p_1 p_3$, and $Y_5 = q_3^2 - p_3^2$. Then, the agent selects option **q** over option **p** if

$$\varepsilon > \beta_1 Y_1 + \beta_2 Y_2 + \beta_3 Y_3 + \beta_4 Y_4 + \beta_5 Y_5. \tag{6}$$

Because the quadratic form may include approximation errors, the residual need not have zero mean. Correspondingly, we include a constant term in the regressions reported below.

Before proceeding to a discussion of the estimation econometric results, we briefly discuss the special case where V is linear in the probabilities. In this case, indifference curves correspond to iso-expected utility curves. The slope of these curves is

$$dp_3/dp_1 = -\beta_2/\beta_1. \tag{7}$$

Recalling eq. (3), we see that the coefficients β_1 and β_2 may be interpreted as differences in von Neumann – Morgenstern utilities at the various wealth levels. In particular, $\beta_1 = -[u(y_2) - u(y_1)]$ and $\beta_2 = u(y_3) - u(y_2)$; we therefore expect $\beta_1 < 0 < \beta_2$. If the agent is risk-neutral, these differences are proportional to the differences in wealth. In our design, $y_1 = -80$, $y_2 = -30$, and $y_3 = 0$. Thus, we define the statistic

$$R = 5\beta_2 + 3\beta_1. \tag{8}$$

For a risk neutral agent, R = 0. If the agent is risk-averse, so that $u(\bullet)$ is concave, then R < 0. Alternatively, R > 0 for a risk-lover. Thus, for an expected-utility maximizer, we may obtain information concerning the agent's risk attitudes from a test of the hypothesis that R = 0. Since the parameters may reflect risk attitudes, we must anticipate differences across agents, and so must perform separate regressions for each subject.

The results reported in Table 1 below are based on a PROBIT regression model, i.e., the disturbance is assumed to follow a normal distribution. (Qualitatively similar results emerged from LOGIT regressions). Here we report parameter estimates and standard errors (given below the corresponding parameter estimate), for each of the 46 individuals for whom the PROBIT regression converged.[2] Also included is the log-likelihood statistic, in the column labeled $\ln L_2$. Broadly speaking, we observe that the first-order effects are more important than second-order effects. Nevertheless, it is evident that non-linear effects are important for a number of subjects. Perhaps because of the limited number of observations, we do not typically observe significance of more than one non-linear effect. Of these, the quadratic effect from the odds of the worst event or the interaction between probabilities of best and worst outcomes seem to be the more important effects. In further results discussed below, we investigate the special cases where only Y_4 is added to the linear terms.

Our primary goal is to determine whether agents' behavior is satisfactorily described by the expected utility hypothesis. This need not mean that agents purposefully act so as to maximize the weighted average of some utility function over wealth; rather, it means that the pattern of choices they exhibit cannot be statistically separated from those an expected utility maximizing agent would make. That is, one cannot reject the hypothesis that $V_k(\bullet)$ is linear. For the specification in eqs. (5) – (6), this implies the parameter restriction $\beta_3 = \beta_4 = \beta_5 = 0$; it also requires that the intercept be zero.[3] To this end, we obtain regression results for the restricted model. Table 2 contains the parameter estimates for the linear model, along with standard errors, in columns 2 and 3. The log-likelihood statistic for the linear model is presented in the sixth column, and the corresponding statistic for the quadratic model is reproduced from Table 1 in the seventh column (these are the columns labeled $\ln L_1$ and $\ln L_2$). We also report the test statistic for the linear restriction on the parameters (in the column labeled χ^2_N). The main result we observe here is the significance of this statistic for a substantial proportion of our subjects – half of the 46 subjects, at the 10% level.[4] This indicates an important divergence from the expected utility model for many of our subjects.

Table 2 also includes estimated values of the statistic R (from eq. 8), along with the test statistic for the hypothesis that R = 0 (presented in the column labeled χ^2_R). For eight subjects, the estimated coefficients β_1 and β_2 had the same sign. Such a representation would imply either that the subject either regarded increased values of the probability of the worst event, or decreased probability of the best event, with favor. Accordingly, we do not compute R for these agents. Of the remaining 38 subjects, 12 had significantly positive values of R and 10 had significantly negative values of R (at the 10% level); these values are consistent with risk-loving and risk-averse behavior, respectively. The estimate for R did not differ significantly from zero for the remaining 16 agents, consistent with risk-neutral behavior. Broadly speaking, these results are inconsistent with a view that agents are typically risk-loving with respect to losses. In addition, we note that the hypothesis of linearity in the representation was rejected for 6 of the 12 apparent risk-lovers. This result suggests the potential for non-expected utility maximizing behavior to be confused with risk-loving behavior.

Next we consider the form of preferences under the non-linear representation. To this end, we perform further regression analysis using a simplified variant of the non-linear representation. While we reject the linear representation for 23 of our subjects, the results in Table 1 indicate a lack of individual significance for any one of the non-linear components, for the majority of our subjects. One likely explanation for this apparent conflict is that the various regressors are quite likely to be collinear. Thus, a more parsimonious representation may produce statistically significant effects from the non-linear term. The simplified representation we employ is

$$V(\mathbf{p}) = \alpha + \beta_1 p_1 + \beta_2 p_3 + \beta_4 p_1 p_3. \tag{9}$$

This form has the advantage of including higher-order effects from both probabilities; it also implies that the slope of an indifference curve

$$dp_3/dp_1 = -(\partial V/\partial p_1)\div(\partial V/\partial p_3) = -[\beta_1 + \beta_4 p_3]/[\beta_2 + \beta_4 p_1], \tag{10}$$

changes with either probability. One expects that increases in p_1 or decreases in p_3 should reduce the value taken by the representation, i.e. $\partial V/\partial p_1 < 0 < \partial V/\partial p_3$. If both inequalities hold, the agent's indifference curves are upward sloping. Note also the curvature of the indifference curves may be described by

$$d^2 p_3/dp_1^2 = 2\beta_4(dp_3/dp_1)/[\beta_2 + \beta_4 p_1]. \tag{11}$$

Thus, if indifference curves are upward sloping and $\partial V/\partial p_3 > 0$, then indifference curves are convex (concave) if $\beta_4 < 0$ ($\beta_4 > 0$).

Table 3 presents the results from the simplified non-linear regression. As in the earlier tables, we present parameter estimates and associated standard errors, along with log-likelihood functions, for each of the subjects. For reference, statistically significant estimates of β_4 are identified. While only 13 of 46 subjects have statistically significant values of β_4, we emphasize that 23 subjects failed the restriction to a linear representation. We note also that the estimate of β_4 is positive for 10 of the 13 significant cases.

In Table 3 we also compute the critical value of p_1 where $\partial V/\partial p_3 = 0$ (presented in the column labeled $\tilde{p}_1$) and the critical value of p_3 where $\partial V/\partial p_1 = 0$ (the column labeled $\tilde{p}_3$). When $\beta_4 < 0$, indifference curves are convex when $p_1 < \tilde{p}_1$ and $p_3 > \tilde{p}_3$. Likewise, when $\beta_4 > 0$, indifference curves are concave when $p_1 > \tilde{p}_1$ and $p_3 < \tilde{p}_3$. This information is summarized in the final column, labeled "characteristic," for those subjects whose choices indicated a rejection of expected utility. For such subjects, the characteristic is "NEU" (non-expected utility), along with the appropriate curvature statement. For some NEU subjects, the curvature is valid over the entire range of probabilities, or over the range used in the experiment ($0 \le p_1 \le .2$; $0 \le p_3 \le .8$). For others, the restrictions on either p_1 or p_3 impinge on a large range of the probabilities used in the experiment. For such individuals, we conclude that choices are inconsistent with EU, but also imply downward sloping indifference curves over a substantial range of the probabilities used in the experiment. We therefore characterize these subjects as "NEU, fails dominance." Similarly, we characterize those agents whose choices fail to reject linear indifference curves, but for whom the parameter estimates imply downward sloping indifference curves, as "EU, fails dominance." The remaining subjects' choices are consistent with the expected utility model. These subjects are identified as "EU;" we also indicate the apparent risk attitude, on the basis of the test of risk neutrality reported in Table 2. Subjects are labeled as either "RA" (risk averse), "RN" (risk neutral), or "RL" (risk loving).

Table 4 summarizes these characteristics. The most notable feature of these results is the overall importance of concave, non-linear indifference curves. Such curves are consistent with "fanning in" in the relevant range for our experiment (though they might be interpreted as "fanning out" in the region where p_1 is large and p_3 small). Concave indifference curves supports Starmer's (2000) caution that the less restrictive "betweenness" axiom still does connect theory with behavior (also see Camerer and Ho, 1994).[5] Our evidence supports the idea that a more descriptive theory would include mixed fanning with nonlinear indifference curves like quadratic utility or models with decision weights.

Our final task is to procure a representation that might be said to summarize our subjects' preferences. To this end, we undertake two additional analyses. First, we analyze behavior of the median voter for each pair-wise comparison. The choices made by the median voter are regressed on the variates Y_1, Y_2, Y_4, and a constant. The results from this regression are reported in the final two rows of Table 3. We see that the coefficients β_1, β_2, and β_4 are each statistically and numerically significant. Furthermore, the estimated representation is globally concave, with the desired properties that $\partial V/\partial p_1 < 0 < \partial V/\partial p_3$.

To elucidate these results, we offer the following thought experiment. Consider two lotteries: lottery *A* sets $p_2 = 1$, and lottery *B* puts $p_1 = 0.25$ and $p_3 = 0.75$. These two lotteries have the same expected value (-\$20), so movements from the first towards the second represent mean-preserving spreads. The impact on the median voter's representation as one moves along this path may be described as

$$dV = [\partial V/\partial p_1 + 3\partial V/\partial p_3]dp_1. \tag{12}$$

Since dp_1 is positive for such movements, it follows that the median voter is made worse of by a mean-preserving spread when

$$\Delta = \beta_1 + 3\beta_2 + \beta_4(p_3 + 3p_1) \tag{13}$$

is negative. Alternatively, if $\Delta > 0$, an increase in risk makes the median voter better off. For a specified lottery, the impact of a mean preserving spread on the median voter's well-being can be empirically identified. We are interested in two hypotheses. The first is that

$$\Delta_A = \beta_1 + 3\beta_2 \tag{14}$$

equals zero. Based on the estimates reported in Table 3, the induced value of Δ_A is -104.06; one may reject the null hypothesis at a probability slightly larger than 11%. We conclude that weak evidence exists that the median voter is worsened by a mean-preserving spread, starting from a point where there is a certain loss of $20. The second hypothesis is that

$$\Delta_B = \Delta_A + 1.5\beta_4 \tag{15}$$

equals zero. Based on the estimates reported in Table 3, the induced value of Δ_B is 235.271; this is significant at about the 4% level. We find that median voter is made better off by a mean-preserving spread, starting from a gamble that places almost all the weight on the two extreme events, and that has an expected loss of $20.

While the results reported from this first analysis are useful, each of the regressions suffers from relatively small degrees of freedom. One could resolve this difficulty by replicating the experiment with many more binary comparisons, although our view is that such an experiment runs a considerable risk that subjects would become fatigued or careless. An alternative approach is to regard each individual's taste parameters (the coefficients in our regression) as drawn from a population; this approach is termed "mixed Logit" (Revelt and Train, 1997; Train, 1998). Under the mixed Logit approach, one identifies the sample mean of the coefficient vector. This mean vector would then provide the summary information for the cohort. We therefore conduct a mixed logit analysis as a second means to identifying behavior of a typical subject.

One obvious advantage of the mixed Logit approach is that the entire dataset may be used in the estimation procedure. There is a dramatic increase in the number of available observations, so much that we can expand the list of explanatory variables without dramatically reducing the degrees of freedom. When we implemented the mixed Logit approach we considered a third order Taylor's series approximation, in which we take the preference representation to be

$$V(\mathbf{p}) = \alpha + \beta_1 p_1 + \beta_2 p_3 + \beta_3 p_1^2 + \beta_4 p_1 p_3 + \beta_5 p_3^2 + \beta_6 p_1^3 + \beta_7 p_1^2 p_3 + \beta_8 p_1 p_3^2 + \beta_9 p_3^3. \quad (12)$$

Based on this specification, the agent prefers lottery **p** over lottery **q** if

$$\varepsilon > \beta_1 Y_1 + \beta_2 Y_2 + \beta_3 Y_3 + \beta_4 Y_4 + \beta_5 Y_5 + \beta_6 Y_6 + \beta_7 Y_7 + \beta_8 Y_8 + \beta_9 Y_9, \quad (13)$$

where Y_1 through Y_5 are as above, and $Y_6 = q_1^3 - p_1^3$, $Y_7 = q_1^2 q_3 - p_1^2 p_3$, $Y_8 = q_1 q_3^2 - p_1 p_3^2$, and $Y_5 = q_3^3 - p_3^3$.

Each agent's tastes are summarized by the vector (β_1, …, β_9), which regard as a draw from a multi-variate distribution. Once the distribution for this vector is specified,[6] the joint likelihood function may be written down. This likelihood function depends on the first two sample moments of the distribution over the parameters, and the stipulated distribution over the error term (e.g., extreme value for the Logit application). Estimates of the mean parameter vector are then obtained via maximum likelihood estimation.

Unfortunately, exact maximum likelihood estimation is not generally possible (Revelt and Train, 1997; Train, 1998). The alternative is to numerically simulate the distribution over the parameters, use the simulated distribution to approximate the true likelihood function, and to then maximize the simulated likelihood function.[7] Table 5 shows the results from such a procedure. Of particular importance here is the observation that the coefficients on all the non-linear terms are statistically important.

We would also like to know if the results are economically important. We used the estimated average parameter vector (the estimates reported in column 2 of Table 5) to numerically identify probability combinations that yield the same value of the value function V(**p**). These combinations can then be used to plot level curves for a "typical" subject within the Marschak-Machina triangle, which we do in Figure 2. The most significant attribute of this diagram is the dramatic non-linearity in the level curves in the heart of the triangle.

These non-linearities are important from a policy perspective. Imagine the present situation implies a lottery such as the one we have marked as A in Figure 2. Consider now a policy that would reduce the chance of the worst loss, event 1, from p_1^0 to p_1^1. Within the context of the Marschak-Machina triangle, an agent with level curves such as we have plotted would then be willing to accept a reduction in the probability of the good event (no loss) from p_3^0 to p_3^1. But an analyst who believed the agent to be (at least approximately) an expected utility maximizer would predict that the agent's level curve was close to the tangent line at lottery A. The analyst would predict that the agent would only accept a reduction from p_3^0 to p_3^2—a dramatic underestimate of the agent's willingness to pay (at least in terms of lower probability of no loss) to reduce the chance of the worst event.

5. Discussion

While our results do not contradict the view that many people who must make choices under uncertainty do not behave in accordance with the expected utility hypothesis, the economic significance of these departures is less clear. A key question is: what approximation errors must be accepted if one is to retain the expected utility model?

Answering this question suggests one should investigate the tendency for the expected utility model to make bad predictions about choices under uncertainty. For example, the expected utility framework suggests that net expected benefits are the appropriate measure of a policy response to a potential environmental catastrophe. If the appropriate measure were based upon a different value function, one that was non-linear in the probabilities, might a different policy be suggested? And if so, what would be the cost of the incorrect action? For one to conclude that the expected utility approach cannot be adequately applied in issues of environmental economics, an evaluation based upon the appropriate value function would have to show the potential for important policy errors, with attendant non-trivial opportunity costs to society.

Consider a scenario in which a decision-maker initially faces substantial risk. Suppose there are two possible outcomes: no loss or very large loss. Such a combination corresponds to $p_2 = 0$ in our framework. Now imagine that an "insurance contract" of sorts is available, one that reduces the chance for the best event, but that also lowers the chance of the worst event. Suppose also that such an arrangement lies below the tangent line to the indifference curve at the initial lottery. Under expected utility, such a policy would be regarded as unambiguously bad.

But if the agent has concave indifference curves – as with many of our subjects, and the median voter – it is entirely possible that such a policy leads to an improvement in well-being. Similar conclusions emerge if the agent's indifference curves are locally concave, as with the model implied by our mixed logit results. Placing this story in the context of a potential environmental disaster, our results suggest the potential for environmental safeguards to raise societal well-being, to the extent that such well-being may be measured by an approach like the median voter model, even when those safeguards have negative net expected benefits.

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TABLE 1: Regression results, quadratic utility model

Subject	β_0	β_1	β_2	β_3	β_4	β_5	LnL_2
1	3.422	48.444	-0.326	-32.277	-60.593	19.503	-8.742
	18.404	736.05	10.566	2454.2	28.953	12.611	
2	0.202	-27.117	-1.878	104.643	-22.346	16.752	-17.584
	0.345	17.143	6.359	70.419	22.601	9.760	
3	0.283	16.384	19.781	-28.853	-20.980	-6.828	-14.179
	0.383	18.904	16.504	72.547	21.300	15.068	
4	-1.099	48.109	4.584	-217.821	3.749	5.141	-10.476
	0.874	31.211	10.161	139.36	21.636	12.493	
6	0.679	-54.813	-15.327	240.10	-19.188	15.396	-9.255
	0.618	23.331	11.989	101.33	24.101	12.622	
7	-0.495	17.582	3.938	-157.23	15.957	-4.010	-20.745
	0.305	14.654	6.183	59.976	22.376	7.258	
8	0.017	-19.239	12.181	40.101	5.108	-3.733	-20.678
	0.317	15.977	7.961	61.705	19.334	8.518	
12	-0.088	-20.797	8.599	79.076	-15.152	-0.048	-21.528
	0.304	15.277	6.445	60.689	18.396	7.511	
13	0.480	-70.331	3.116	170.38	46.382	-12.138	-8.207
	0.608	28.260	11.162	109.96	37.923	13.603	
14	0.531	-53.046	26.690	131.25	16.976	-14.744	-14.537
	0.392	21.277	11.368	76.869	30.702	12.369	
15	0.293	-46.552	14.435	144.32	4.825	-15.793	-13.652
	0.400	19.556	7.395	73.058	25.600	8.913	
16	0.136	37.418	-5.943	-154.02	0.450	9.543	-18.512
	0.348	16.618	6.813	60.315	17.874	8.456	
17	-0.224	-7.854	11.516	36.193	-9.202	-5.488	-23.183
	0.300	14.403	7.625	55.324	17.346	7.820	
18	-0.296	-18.284	28.355	-0.008	8.474	-11.369	-14.441
	0.377	19.981	17.552	68.311	27.523	17.916	
19	-0.195	-53.719	10.723	14.011	53.162	5.750	-11.713
	0.382	23.571	9.750	86.456	47.228	15.292	
20	0.331	-19.059	1.085	-0.480	20.847	-0.794	-19.669
	0.303	16.688	6.453	60.492	27.977	7.862	
21	-0.345	-19.299	11.094	-26.734	31.284	-9.500	-22.272
	0.296	15.425	6.891	56.308	24.295	7.733	
22	-0.043	30.046	-8.031	-294.49	-141.96	101.77	-5.409
	0.560	42.392	19.432	233.72	142.03	65.900	
23	0.469	-80.263	15.098	274.52	11.389	-6.246	-14.761
	0.430	23.369	7.031	92.664	20.150	7.840	
24	0.291	-40.158	-4.375	5.725	53.411	4.409	-16.561
	0.330	23.156	9.243	76.485	48.664	11.769	
25	1.251	-37.879	4.351	18.347	42.532	5.623	-10.851
	0.442	26.472	10.128	95.834	52.542	13.984	
26	6.644	-2624.5	404.39	-3153.1	4242.8	-424.28	-1.910
	91.26	29359.0	4442.5	47834.0	48938.0	4893.9	
27	-0.571	18.562	4.116	-122.922	7.661	0.511	-20.501
	0.353	15.637	6.846	59.286	18.850	8.133	
28	-0.167	15.034	-7.171	-48.298	-3.689	8.978	-24.535
	0.296	14.190	6.106	53.231	17.089	7.037	
29	-0.290	-40.176	5.149	30.532	16.747	24.256	-11.310

	0.409	23.410	9.863	79.919	42.894	19.051	
31	-0.925	-13.355	2.803	29.082	-4.525	0.721	-21.984
	0.329	15.480	6.303	57.035	16.452	6.953	
32	0.059	-57.927	3.273	128.05	0.798	18.256	-10.640
	0.394	27.370	8.594	97.404	54.016	15.351	
33	0.448	-2.014	1.696	-56.030	-76.072	46.760	-8.133
	0.474	30.128	16.089	132.41	96.546	36.252	
34	10.576	68.318	-335.21	1383.8	-1884.0	1610.1	-0.00003
	1058.1	37136.0	3609.7	176200.0	20415.0	15216.0	
35	20.778	-5670.2	827.07	-954.15	8589.8	-966.49	-0.00004
	207.2	49771.0	7238.9	36761.0	76581.0	8691.1	
36	2.900	-144.70	9.945	702.88	-64.536	-6.346	-5.557
	5.469	185.16	10.315	882.11	48.920	17.905	
37	-0.204	9.896	6.627	-42.516	-1.797	-6.384	-25.805
	0.290	13.529	6.169	51.445	16.257	6.825	
38	-1.366	-15.698	7.830	12.893	-82.043	33.363	-8.847
	0.557	24.523	9.845	96.290	66.711	23.304	
39	-0.757	-188.25	42.896	638.06	27.388	-1.715	-5.632
	0.824	69.661	19.788	249.37	32.528	20.467	
40	11.779	-2806.02	400.10	-2286.0	4249.4	-424.94	-1.910
	144.7	31639.0	4486.8	39641.0	49690.0	4969.1	
41	0.181	-5.701	-6.142	113.595	-32.323	10.782	-22.574
	0.286	14.495	6.653	57.019	22.543	7.939	
42	0.512	-40.707	17.925	-107.33	86.085	-12.774	-12.529
	0.380	27.008	13.854	110.35	63.479	16.718	
43	0.379	68.487	-9.241	-192.96	-25.728	14.523	-11.532
	0.517	27.390	9.030	90.270	24.798	10.712	
44	0.021	48.142	-10.269	-174.24	-147.12	47.973	-9.914
	4.044	224.72	10.042	2010.9	94.620	26.175	
45	0.379	-85.060	15.897	58.960	83.775	-9.029	-11.287
	0.403	34.141	9.826	128.58	73.905	13.464	
46	0.137	-42.848	8.651	119.02	21.003	-6.735	-21.794
	0.309	15.211	5.975	55.459	18.129	6.917	
47	-0.223	28.045	24.529	-124.67	-15.414	3.430	-9.051
	0.537	24.291	32.735	88.280	32.713	38.713	
48	-0.157	-24.365	-17.052	-192.71	3.900	34.941	-6.530
	3.370	189.08	19.763	1680.0	117.58	28.890	
49	0.243	-139.07	11.001	60.087	138.85	2.012	-5.686
	0.558	84.145	17.036	209.46	166.31	24.677	
51	-0.047	-13.063	8.717	-41.748	26.520	-0.355	-17.838
	0.330	17.488	8.889	64.373	28.534	11.395	
53	-0.321	3.696	-0.314	-23.150	-15.845	2.948	-23.996
	0.283	14.057	5.692	52.425	17.766	6.315	

TABLE 2: Regression results, linear utility model

Subject	β_1	β_2	R	χ^2_R	LnL_1	LnL_2	χ^2_N
1	-5.292	5.362	10.934	1.51	-23.458	-8.742	29.432**
	-2.958	-2.224					
2	-10.084	6.286	1.181	0.02	-20.794	-17.584	6.42
	3.386	2.484					
3	-1.818	11.157	50.332	9.53**	-16.957	-14.179	5.556
	3.408	4.199					
4	4.354	10.306	%	n.a.	-13.198	-10.476	5.444
	3.778	4.537					
6	-14.973	-0.770	&	n.a.	-15.075	-9.255	11.64**
	4.488	1.572					
7	-6.256	1.486	-11.337	1.86	-25.104	-20.745	8.718+
	2.893	1.450					
8	-6.879	7.931	19.017	3.38+	-21.034	-20.678	0.712
	3.290	2.819					
12	-7.171	5.150	4.238	0.23	-23.025	-21.528	2.994
	3.054	2.175					
13	-10.420	-4.619	&	n.a.	-12.674	-8.207	8.934+
	4.435	2.538					
14	-13.676	9.672	7.333	0.51	-18.025	-14.537	6.976
	4.438	3.249					
15	-11.780	-0.760	&	n.a.	-18.056	-13.652	8.808+
	3.860	1.541					
16	-0.486	4.250	19.791	5.59*	-23.471	-18.512	9.918*
	2.608	1.733					
17	-1.100	3.945	16.427	3.6+	-24.719	-23.182	3.074
	2.719	1.966					
18	-11.608	16.945	49.898	1.78	-15.038	-14.441	1.194
	4.492	5.484					
19	-15.350	12.386	15.881	5.21*	-16.636	-11.713	9.846*
	5.073	4.122					
20	-11.255	2.344	-22.046	0.17	-20.799	-19.669	2.26
	3.507	1.598					
21	-6.740	3.356	-3.441	2.99+	-24.132	-22.272	3.72
	2.924	1.626					
22	-39.225	29.471	29.681	0.49	-10.013	-5.409	9.208+
	15.256	11.257					
23	-8.708	4.010	-6.073	4.60*	-22.505	-14.761	15.488**
	3.016	1.945					
24	-9.848	2.117	-18.958	0.07	-22.040	-16.561	10.958*
	3.291	1.574					
25	-13.740	8.783	2.697	3.97*	-18.337	-10.851	14.972**
	4.409	3.136					
26	-99.570	24.364	-176.891	3.49+	-4.480	-1.910	5.14
	48.493	11.823					
27	-2.663	4.772	15.870	2.96+	-23.336	-20.501	5.67
	2.668	1.775					
28	2.694	1.129	%	n.a.	-26.188	-24.535	3.306
	2.585	1.474					
29	-13.897	14.349	30.056	4.9*	-16.066	-11.310	9.512*

	4.896	4.571					
31	-0.858	0.473	-0.211	0.01	-27.649	-21.984	11.330**
	2.516	1.431					
32	-17.446	9.562	-4.524	0.18	-16.416	-10.640	11.552**
	5.440	3.269					
33	-33.308	24.009	20.122	1.58	-11.063	-8.133	5.86
	12.589	9.304					
34	-7.751	12.967	41.585	9.34**	-16.081	-.00003	32.162**
	3.670	3.690					
35	-21.691	5.540	-37.373	8.06**	-14.121	-.00004	28.242**
	6.416	2.721					
36	-15.880	0.021	-47.533	13.83**	-15.194	-5.557	19.274**
	4.581	1.580					
37	1.044	0.069	%	n.a.	-27.602	-25.805	3.594
	2.513	1.439					
38	-21.066	13.438	3.992	0.11	-14.721	-8.847	11.748**
	6.959	4.781					
39	-5.057	8.729	28.477	5.74*	-20.520	-5.632	29.776**
	3.247	3.189					
40	-85.641	16.785	-173.001	3.97*	-4.304	-1.910	4.788
	46.618	11.120					
41	3.762	0.645	%	n.a.	-25.921	-22.574	6.694
	2.708	1.470					
42	-17.833	11.474	3.870	0.12	-15.809	-12.529	6.56
	5.443	4.020					
43	6.055	5.111	%	n.a.	-16.557	-11.532	10.05*
	3.368	2.389					
44	-23.493	11.151	-14.722	1.16	-14.051	-9.914	8.274+
	7.530	3.897					
45	-20.974	8.062	-22.613	3.46+	-15.050	-11.287	7.526
	6.595	3.298					
46	-5.371	2.656	-2.831	0.13	-25.280	-21.795	6.97
	2.720	1.571					
47	-7.602	28.380	119.095	8.51**	-10.547	-9.051	2.992
	4.988	10.319					
48	-34.419	8.496	-60.777	7.59*	-11.552	-6.530	10.044*
	11.661	3.881					
49	-21.163	6.868	-29.146	5.41*	-14.755	-5.686	18.138**
	6.534	3.084					
51	-9.547	10.528	24.001	4.10*	-19.007	-17.838	2.338
	3.793	3.533					
53	-5.976	0.611	-14.876	3.18+	-24.994	-24.000	1.988
	2.778	1.472					

+: significant at 10% level or better
*: significant at 5% level or better
**: significant at 1% level or better

TABLE 3: Regression results, simple non-linear utility model

Subject	β_0	β_1	β_2	β_4	lnL	$\tilde{p}_1$	$\tilde{p}_3$	characteristic
1	2.765	23.217	13.557	-28.616+	-10.200	0.4738	0.8113	NEU
	1.062	10.64	5.957	17.062				convex[a,b]
2	-0.071	-15.746	5.598	11.046	-20.342	-0.5068	1.4255	EU, RN
	0.278	7.051	2.738	13.376				
3	0.338	13.42	14.471	-29.495+	-14.386	0.4906	0.4550	EU, RL
	0.350	7.86	5.249	15.977				
4	-0.246	-0.078	11.489	5.999	-12.846	-1.9152	0.0130	EU, fails
	0.345	9.084	5.465	19.148				dominance
6	-0.023	-19.814	-1.888	12.426	-14.756	0.1519	1.5946	NEU, fails
	0.311	7.694	2.487	16.067				dominance
7	-0.170	-5.691	2.333	-4.365	-24.866	0.5345	-1.3038	NEU,
	0.263	5.928	2.062	11.761				convex[a]
8	-0.043	-7.779	7.899	1.269	-21.013	-6.2246	6.1300	EU, RL
	0.281	6.915	3.074	13.515				
12	-0.239	-4.411	6.621	-10.530	-22.425	0.6288	-0.4189	EU, RN
	0.274	6.211	2.598	12.644				
13	0.117	-32.517	-9.942	43.51*	-10.047	0.2285	0.7474	NEU, fails
	0.381	11.955	4.394	21.41				dominance
14	0.199	-10.174	9.919	-4.737	-17.675	2.0939	-2.1478	EU, RN
	0.290	8.046	3.722	15.536				
15	-0.083	-2.654	1.250	-24.342	-17.130	0.0514	-0.1090	NEU, fails
	0.295	8.116	2.444	19.492				dominance
16	0.346	-2.772	2.896	10.866	-22.525	-0.2665	0.2551	NEU
	0.287	6.580	2.046	13.059				concave[d]
17	-0.264	3.912	6.060	-15.856	-23.632	0.3822	0.2467	EU, RL
	0.273	6.016	2.501	12.267				
18	-0.289	-11.804	18.845	-5.593	-14.648	3.3694	-2.1105	EU, RN
	0.340	8.313	6.459	16.505				
19	-0.216	-57.038	13.901	66.726*	-11.785	-0.2083	0.8548	NEU
	0.339	19.305	5.065	29.865				concave[d]
20	0.333	-18.037	0.471	18.709	-19.674	-0.0252	0.9641	EU, RN
	0.278	9.364	2.096	16.698				
21	-0.274	-12.609	3.461	7.753	-23.243	-0.4464	1.6263	EU, RA
	0.272	6.854	2.131	13.259				
22	0.461	-51.414	29.846	23.596	-9.000	-1.2649	2.1789	NEU
	0.394	21.651	12.464	25.400				concave
23	-0.054	-14.622	3.354	11.42	-22.038	-0.2938	1.2809	NEU
	0.275	7.095	2.297	13.284				concave
24	0.288	-46.781	-0.979	67.295**	-16.637	0.0145	0.6952	NEU
	0.313	16.615	2.299	26.571				concave[c,d]
25	1.217	-43.191	7.940	60.003*	-10.932	-0.1323	0.7198	NEU
	0.393	19.952	4.269	30.666				concave
26	-0.355	-290.890	53.290	103.87	-3.040	-0.5130	2.8005	EU, RA
	10.410	2079.0	467.00	92.660				
27	-0.236	-6.630	5.160	4.447	-22.832	-1.1603	1.4909	EU, RL
	0.280	6.678	2.098	12.856				
28	-0.085	-2.422	0.600	9.892	-25.712	-0.0607	0.2448	EU, fails
	0.265	5.940	1.946	11.964				dominance

29	-0.342	-49.783	17.048	57.775+	-12.220	-0.2951	0.8617	NEU
	0.346	19.364	6.145	29.918				concave[d]
31	-0.966	-7.939	3.130	-1.907	-22.132	1.6413	-4.1631	NEU
	0.319	5.860	2.086	11.273				convex
32	-0.105	-64.810	10.821	71.81*	-11.832	-0.1507	0.9025	NEU
	0.342	23.362	4.069	33.309				concave[d]
33	0.600	-45.378	25.466	21.628	-9.535	-1.1775	2.0981	EU, RN
	0.382	19.871	11.255	24.397				
34	0.070	-17.376	12.376	20.936	-15.320	-0.5911	0.8300	NEU
	0.320	9.779	4.094	18.093				concave[d]
35	0.071	-109.031	8.379	114.816**	-6.912	-0.0730	0.9496	NEU
	0.475	41.237	5.332	47.104				concave[d]
36	-0.242	-1.100	5.320	-55.155+	-12.76	0.0965	-0.0199	NEU
	0.350	9.782	3.558	31.581				convex[a]
37	-0.095	6.891	1.635	-14.827	-26.684	0.1103	0.4648	EU, fails
	0.266	5.505	1.916	11.040				dominance
38	-1.145	-38.406	20.609	1.807	-10.213	-11.4051	21.2540	NEU
	0.467	17.672	7.312	21.509				concave
39	-0.704	-22.45	10.700	24.224	-16.341	-0.4417	0.9267	NEU
	0.343	9.572	4.000	18.151				concave[d]
40	-0.150	-267.80	42.300	103.86	-3.040	-0.4073	2.5785	EU, RA
	10.040	2003.0	451.00	92.660				
41	-0.053	4.004	0.910	-1.490	-27.274	0.6107	2.6872	EU, fails
	0.263	5.894	2.004	11.768				dominance
42	0.669	-34.353	10.657	33.156	-13.151	-0.3214	1.0361	EU, RN
	0.337	15.813	4.537	22.676				
43	0.366	13.417	4.957	-9.59	-15.592	0.5170	1.3994	NEU
	0.326	8.338	2.774	15.991				convex[a]
44	0.324	-22.346	10.477	1.882	-13.427	-5.5670	11.8735	NEU
	0.306	14.716	4.001	24.316				concave
45	0.235	-62.145	7.964	63.081*	-11.992	-0.1263	0.9852	EU, RA
	0.347	22.416	3.892	29.965				
46	-0.052	-11.909	1.919	12.928	-24.609	-0.1484	0.9212	EU, RN
	0.268	6.501	1.962	12.408				
47	0.111	-0.022	30.959	-16.959	-10.135	1.8255	-0.0013	EU, RL
	0.424	9.600	11.578	19.850				
48	0.302	-88.210	8.090	87.459*	-8.603	-0.0925	1.0086	NEU
	0.433	33.486	4.732	41.479				concave
49	0.202	-143.98	11.812	162.89**	-5.727	-0.0725	0.8839	NEU
	0.505	56.740	6.515	70.380				concave[d]
51	0.054	-20.635	9.502	22.679	-18.053	-0.4190	0.9099	EU, RL
	0.290	10.155	3.740	17.675				
53	-0.289	-3.692	2.345	-11.066	-24.175	0.2119	-0.3336	EU, RA
	0.268	5.816	2.097	12.597				
Median	-.943	-269.03*	54.992*	226.219*	-4.384	-.243	1.189	NEU
voter	1.178	126.95	27.031	114.655				concave

Notes: +: significant at 10% level; *: significant at 5% level; **: significant at 10% level

a: if $p_1 < \tilde{p}_1$; b: if $p_3 > \tilde{p}_3$;

c: if $p_1 > \tilde{p}_1$; d: if $p_3 > \tilde{p}_3$

TABLE 4: Characterization of Individual Subjects

characterization	number of subjects	subject IDs
EU, risk-neutral	8	2,12,14,18,20,33,42,46
EU, risk-averse	5	21,26,40,45,53
EU, risk-loving	6	3,8,17,27,47,51
NEU, convex ICs over relevant range	3	7,31,43
NEU, limited convex ICs	2	1,36
NEU, concave ICs over relevant range	13	19,22,23,25,29,32,34,35,38,39,44,48,49
NEU, limited concave ICs	2	16,24
EU, failed dominance	4	4,28,37,41
NEU, failed dominance	3	6,13,15
Choices did not vary enough to allow estimation	7	5,9,10,11,30,50,52

TABLE 5: Results of Mixed LOGIT analysis

regressor	First moment	Second moment
Y_1	11.726	.3350
	(7.298)	(1.055)
Y_2	2.6787	.2609
	(1.810)	(.3453)
Y_3	-349.77	1.549
	(87.65)	(4.873)
Y_4	32.181	4.550
	(16.64)	(2.009)
Y_5	-10.766	.4148
	(4.752)	(.4151)
Y_6	1038.2	1.710
	(331.54)	(17.04)
Y_7	202.97	4.527
	(61.69)	(7.685)
Y_8	-98.239	.1151
	(19.778)	(2.401)
Y_9	16.187	.1178
	(4.656)	(.5003)

Log-likelihood statistic -1360.34

Asymptotic standard errors in parentheses

*: significant at 5% level or better
**: significant at 1% level or better

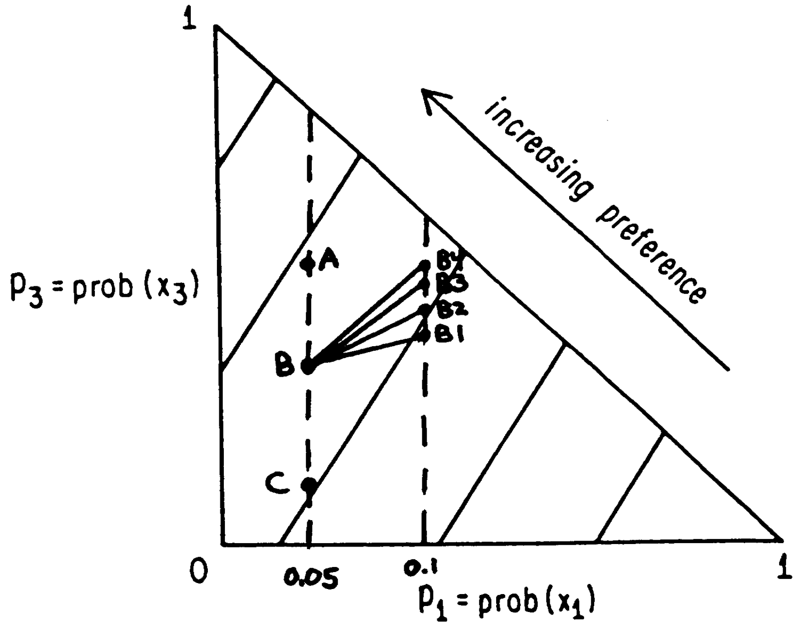

Figure 1: Comparison of Lotteries in Our Experimental Design

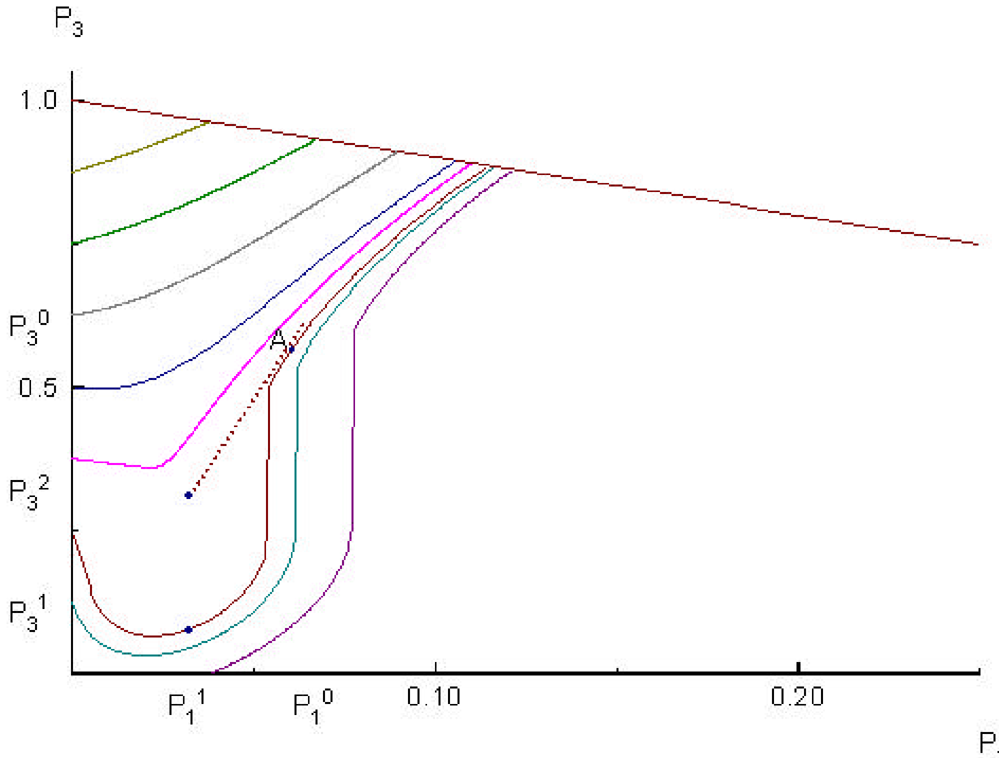

Figure 2: Level curves implied by cubic representation over lotteries

APPENDIX 1: EXPERIMENTAL INSTRUCTIONS

INSTRUCTIONS

Welcome. This is an experiment in decision making that will take about an hour to complete. You will be paid in cash for participating at the end of the experiment. How much you earn depends on your decisions and chance. **Please do not talk** and do not try to communicate with any other subject during the experiment. If you have a question, please raise your hand and a monitor will come over. If you fail to follow these instructions, you will be asked to leave and forfeit any moneys earned. You can leave the experiment at any time without prejudice. Please read these instructions carefully, and then review the answers to the questions on page 4.

AN OVERVIEW: You will be presented with 40 pairs of options. For each pair, you will pick the option you prefer. After you have made all 40 choices, you will then play one of the 40 options to determine your take-home earnings.

THE EXPERIMENT

Stage #1: The Option Sheet: After filling out the waiver and the survey forms, the experiment begins. You start with $100, and your choices and chance affect how much of this money you can keep as your take-home earnings.

You will be given an **option sheet** with 40 pairs of options. For each pair, you will circle the option you prefer. Each option is divided into 3 probabilities:

P1 is the probability you will lose $*80*;

P2 is the probability you will lose $*30*; and

P3 is the probability you will lose $*0*.

For each option, the three probabilities always add up to 100% (P1 + P2 + P3 = 100%). For example, if an option has P1=20%, P2=50% and P3=30%, this implies you have a 20% chance to lose $80, a 50% chance to lose $30, and a 30% chance to lose $0.

On your option sheet, you circle your preferred option for each of the 40 pairs. For example, consider the pair of options, *A* and *B*, presented below. Suppose after examining the pair of options carefully, you prefer option *A* to *B*—then you would circle *A* (as shown below). If you prefer *B*, you would circle *B*.

A	*B*
P1=10%, P2 =20%, P3 =70%	P1=20%, P2 =20%, P3 =60%

Stage #2: The Tan Pitcher: After filling out your option sheet, please wait until the monitor calls you to the front of the room. When called, bring your waiver form, survey, and option sheet with you.

On the front table is a tan pitcher with 40 chips inside, numbered 1 to 40. The number on the chip represents the option you will play from your option sheet. You will reach into the tan pitcher without looking at the chips, and pick out a chip. The number on the chip determines which option you will play to determine your take-home earnings. For example, if you draw chip #23, you will play the option you circled for the pair #23 on your option sheet.

Stage #3: The Blue Pitcher: After you have selected the option you will play, you then draw a different chip from a second pitcher—the blue pitcher. The blue pitcher has 100 chips, numbered 1 to 100. The number on the chip determines the actual outcome of the option—a loss of either $80, $30, or $0.

For example, if your option played has

P1=10%
P2=50%
P3=40%,

then if you pick a chip numbered between 1 and 10, you lose $80; if you pick a chip between 11 and 60, you lose $30; or if you pick a chip between 61 and 100, you lose $0.

If instead, your option played has

P1=20%
P2=20%
P3=60%,

then if you pick a chip between 1 and 20, you lose $80; if you pick a chip between 21 and 40, you lose $30; or if you pick a chip between 41 and 100, you lose $0.

Stage #4: Ending the experiment: After playing the option, you fill out a tax form. The monitor will then hand over your take-home earnings, and you can leave the room.

Now please read through the questions and answers on the next page.

QUESTIONS and ANSWERS

1. When I make a choice, I will choose between how many options?
   2
2. I will make how many choices?
   40
3. My initial $$ endowment is how much?
   $100
4. P1 represents what?
   The probability of losing $80
5. P2 represents what?
   The probability of losing $30
6. P3 represents what?
   The probability of losing $0
7. For each option, the three probabilities sum to what?
   100%
8. What does the number drawn from the tan pitcher represent?
   The option (1 to 40) played from your option sheet
9. What does the number drawn from the blue pitcher represent?
   The outcome (1 to 100) of the option played—determining whether you lose either $80, $30, or $0

Are there any questions?

APPENDIX 2: THE SURVEY SHEET

1. Social Security Number: ______________________________
2. Gender: (circle) Male Female
3. Birthdate: ________________ (month/day/year)
4. Highest Level of School Completed: (please circle)
   - Junior High School
   - High School or Equivalency
   - College or Trade School
   - Graduate or professional School
5. Courses Taken in Mathematics: (please circle all that apply)
   - College Algebra
   - Calculus or Business Calculus
   - Linear Algebra
   - Statistics or Business Statistics
6. Families' Annual Income: ____________________
7. Personal Annual Income: ____________________

THANK YOU

APPENDIX 3: THE OPTION SHEET

Social Security Number: ___________________________

An Example:

A	B
P1=10%, P2 =20%, P3 =70%	P1=20%, P2 =20%, P3 =60%
10% chance of losing $80 20% chance of losing $30 70% chance of losing $0	20% chance of losing $80 20% chance of losing $30 60% chance of losing $0

#	A	B
1.	P1=5%, P2=35%, P3=60%	P1=1%, P2=40%, P3=59%
2.	P1=20%, P2=0%, P3=80%	P1=20%, P2=39%, P3=41%
3.	P1=5%, P2=35%, P3=60%	P1=1%, P2=49%, P3=50%
4.	P1=5%, P2=35%, P3=60%	P1=5%, P2=55%, P3=40%
5.	P1=5%, P2=55%, P3=40%	P1=1%, P2=69%, P3=30%
6.	P1=5%, P2=75%, P3=20%	P1=20%, P2=50%, P3=30%
7.	P1=5%, P2=55%, P3=40%	P1=1%, P2=60%, P3=39%
8.	P1=5%, P2=75%, P3=20%	P1=20%, P2=59%, P3=21%
9.	P1=5%, P2=75%, P3=20%	P1=1%, P2=80%, P3=10%
10.	P1=5%, P2=55%, P3=40%	P1=20%, P2=30%, P3=50%
11.	P1=5%, P2=75%, P3=20%	P1=1%, P2=80%, P3=19%
12.	P1=5%, P2=55%, P3=40%	P1=20%, P2=39%, P3=41%
13.	P1=5%, P2=35%, P3=60%	P1=10%, P2=25%, P3=65%
14.	P1=5%, P2=35%, P3=60%	P1=20%, P2=10%, P3=70%
15.	P1=5%, P2=35%, P3=60%	P1=10%, P2=10%, P3=80%
16.	P1=5%, P2=35%, P3=60%	P1=20%, P2=19%, P3=61%
17.	P1=5%, P2=55%, P3=40%	P1=10%, P2=45%, P3=45%

18. P1=5%, P2=75%, P3=20% P1=10%, P2=60%, P3=30%

19. P1=5%, P2=55%, P3=40% P1=10%, P2=30%, P3=60%

20. P1=5%, P2=75%, P3=20% P1=10%, P2=69%, P3=21%

21. P1=5%, P2=35%, P3=60% P1=1%, P2=44%, P3=55%

22. P1=10%, P2=10%, P3=80% P1=10%, P2=49%, P3=41%

23. P1=5%, P2=35%, P3=60% P1=1%, P2=59%, P3=40%

24. P1=1%, P2=40%, P3=59% P1=1%, P2=79%, P3=20%

25. P1=5%, P2=55%, P3=40% P1=1%, P2=79%, P3=20%

26. P1=5%, P2=75%, P3=20% P1=20%, P2=40%, P3=40%

27. P1=5%, P2=55%, P3=40% P1=1%, P2=64%, P3=35%

28. P1=5%, P2=75%, P3=20% P1=20%, P2=55%, P3=25%

29. P1=5%, P2=75%, P3=20% P1=1%, P2=99%, P3=0%

30. P1=5%, P2=55%, P3=40% P1=20%, P2=20%, P3=60%

31. P1=5%, P2=75%, P3=20% P1=1%, P2=84%, P3=10%

32. P1=5%, P2=55%, P3=40% P1=20%, P2=35%, P3=45%

33. P1=5%, P2=35%, P3=60% P1=10%, P2=29%, P3=61%

34. P1=5%, P2=35%, P3=60% P1=20%, P2=0%, P3=80%

35. P1=5%, P2=35%, P3=60% P1=10%, P2=20%, P3=70%

36. P1=5%, P2=35%, P3=60% P1=20%, P2=10%, P3=65%

37. P1=5%, P2=55%, P3=40% P1=10%, P2=49%, P3=41%

38. P1=5%, P2=75%, P3=20% P1=10%, P2=50%, P3=40%

39. P1=5%, P2=55%, P3=40% P1=10%, P2=40%, P3=50%

40. P1=5%, P2=75%, P3=20% P1=10%, P2=65%, P3=25%

ENDNOTES

1 On the triangle diagram, such behavior is illustrated by a “fanning out” if indifference curves are convex, or a “fanning in” if they are concave. Other examples of the Allais Paradox include the "common consequence effect," "the certainty effect," and the "Bergen Paradox." See Machina (1982, 1987) or Thaler (1992) for a more in depth treatment of these cases and the literature involving independence violations.

2 Of the 53 subjects, 7 made choices that did not vary sufficiently to allow our regressions to converge. Thus, the tables presented in the paper do not include output for subjects 5, 9, 10, 11, 30, 50, and 52.

3 Recall that we interpret the intercept as the result of approximation error. If the linear form is correct, there is no approximation and hence no role for the intercept to play.

4 Since there are four restrictions in this hypothesis, the test statistic (twice the difference in the value of the log-likelihood functions with and without the parameter restriction) would be distributed as a chi-squared variate with 4 degrees of freedom. The critical points are 7.78, 9.49, and 11.1 at the at 10%, 5%, and 1% levels, respectively.

5 Recall the betweenness axiom is a weaken form of the independence axiom. Betweenness says that preferences are such that any probability mixture of two lotteries will be ranked between the two (see Starmer, 2000).

6 To this end, we assumed that the parameter vector was multi-normally distributed.

7 We thank Kenneth Train for supplying the GAUSS program with which we conducted the estimation.